Int. J. Heat Mass Transfer. Vol. 27, No. 11, pp. 2095–2100, 1984
Printed in Great Britain

0017-9310/84 $3.00 + 0.00

Friction factor prediction for fully developed laminar twisted-tape flow

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(*Received* 2 *November* 1983 *and in revised form* 10 *February* 1984)

Abstract—Friction factor characteristics are necessary for the evaluation of the efficiency of heat transfer enhancement by means of a twisted-tape insert in a smooth tube. Effective flow parameters are introduced to facilitate the correlation of friction factor data in such cases. A utility chart from which the friction factor—Reynolds number product for twisted-tape flow may be predicted, is presented in graphical form. Results from a parametric numerical study on laminar, constant property twisted-tape flow are provided and also presented graphically. These results are used to construct a correlative expression from which friction factor characteristics for laminar flow may be obtained. The finite thickness of the tape is taken into consideration.

1. INTRODUCTION

CONVECTIVE heat transfer in a tubular heat exchanger may be enhanced considerably by the introduction of a twisted-tape insert. This static measure, however, adversely affects the pumping power necessary to maintain the mass flow rate. It is therefore of cardinal importance to be able to predict the friction factor for a twisted-tape section of specified geometrical dimensions and fluid properties.

In this paper an attempt is made towards a suitable correlative equation for the friction factor in the case of laminar, constant property flow. Since the entrance length for the flow development is shortened markedly by the presence of the tape [1], only fully developed flow will be considered here. A correlation to this effect and based on numerical work by Date [2] was proposed by Date and Singham [3] and modified by Shah and London [4]. For the present work, use is made of numerical work [5] employing a primitive variable finite-difference solution procedure.

In accordance with a suggestion by Shah and London the friction factors and Reynolds numbers are based on the real inner diameter of the smooth tube. This facilitates comparison with no-tape flow conditions.

2. THE EFFECTIVE FLOW PARAMETERS

An attractive way of correlating experimental data for twisted-tape flow was proposed by Nazmeev and Nikolaev [6]. According to their view the real flow situation should be considered, namely that the fluid is being forced through a non-semicircular channel, helically wound around the tube axis. Effective physical dimensions for the flow may thus be computed, whereby results close to the straight duct results can be obtained. The cross-sectional flow area is depicted in Fig. 1.

The predominant direction of low is along the helical lines brought about by the presence of the tape. Firstly the effective cross-sectional flow area A_ε for the thin tape case with $\gamma = 0$ is obtained by summation of the incremental area elements normal to the helical flow lines at each point

$$A_\varepsilon = 2\int_0^\pi \int_0^a r \sin\alpha \, dr \, d\theta = \frac{2H^2}{\pi}(G-1) \quad (1)$$

where the helical factor G is defined as

$$G \equiv \sqrt{\left(1+\frac{\pi^2 D^2}{4H^2}\right)} = \operatorname{cosec}\alpha. \quad (2)$$

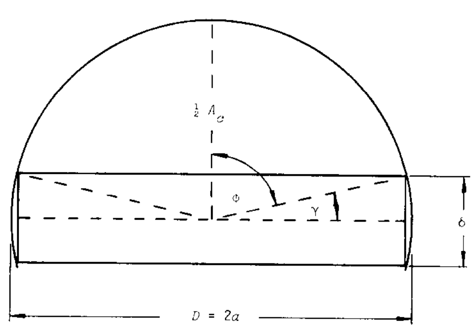

FIG. 1. Cross-sectional segment formed by the presence of a tape of finite thickness δ.

NOMENCLATURE

A	cross-sectional flow area
D	inner diameter of tube
f	apparent Fanning friction factor, $(\Delta\bar{p}/\rho u_m^2)(D/2L)$
$f(\varepsilon)$	friction factor defined through effective flow concept
G	helical factor defined in equation (2)
H	tape pitch for twist of π radians
L	length of tube
p	pressure
P	wetted perimeter
r	radial coordinate
Re	Reynolds number
u	axial velocity
x	axial distance
y	tape twist parameter, H/D.

Greek symbols

α	angle of helicity
γ	half the angle spanned by the tape edge at the tube axis
δ	tape thickness
Δ	increment
θ	angular coordinate
Λ	conversion factor, $D^2A/(D_\varepsilon^2 A_\varepsilon)$
π	3.14159
ρ	fluid density.
∞	denotes fully developed region.

Subscripts

an	analytically derived
c	real flow area
D	based on diameter of tube
m	cross-sectionally mean value
t	tapeless
ε	pertaining to effective flow parameters.

When this result is combined with the case of a tape of finite thickness ($\gamma \neq 0$) it follows for $\delta \ll D$ that

$$A_\varepsilon = \frac{2H^2}{\pi}(G-1) - D\delta. \quad (3)$$

An effective wetted perimeter is also taken into consideration according to Fig. 2, namely

$$P_\varepsilon = 2\left(D - \delta + \int_0^\pi r \sin\alpha \, d\theta\right)$$
$$= 2\left(D - \delta + \frac{\pi D}{2G}\right). \quad (4)$$

The effective mean velocity $u_{m,\varepsilon}$ is defined so as to conserve the mass flow rate indicated by the mean axial velocity u_m

$$u_{m,\varepsilon} \equiv \frac{A_c}{A_\varepsilon} \cdot u_m. \quad (5)$$

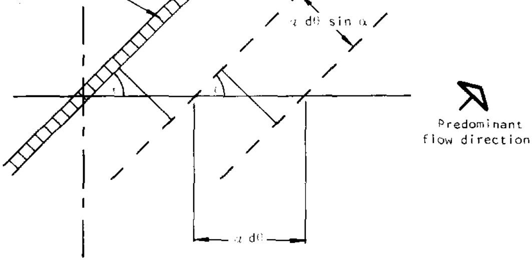

FIG. 2. Effective wetted perimeter along the tube surface. The tube-and-tape assembly is viewed laterally.

Similarly, the effective length of the helical passage is defined so that the overall volume remains the same

$$x_\varepsilon \equiv \frac{A_c}{A_\varepsilon} \cdot x. \quad (6)$$

The effective hydraulic diameter D_ε for the helical passage is then defined as

$$D_\varepsilon \equiv \frac{4A_\varepsilon}{P_\varepsilon}. \quad (7)$$

In 1966, Seymour [7] suggested the use of an effective flow length, but he recommended Gx, which is the length of the helix formed by the tape edge. The present formalism, however, is more acceptable since it takes into account the average length of the idealized helical streamlines.

An interesting point is evident from Table 1 where some of the numerical values for the coefficients are presented for the case of a thin tape when $\delta = 0$. The quotient D/D_ε reaches a minimum at some y-value y_0 which may be obtained as follows for $\delta = 0$

$$\frac{D}{D_\varepsilon} = \frac{(2G+\pi)(G+1)}{2\pi G} \quad (8)$$

with a minimum at

$$G_0 = \sqrt{\left(\frac{\pi}{2}\right)}. \quad (9)$$

Thus

$$y_0 = \frac{\pi}{\sqrt{(2\pi - 4)}} = 2.079 \quad (10)$$

Table 1. Numerical factors for effective flow calculations with $\delta = 0$. Only the last column is restricted to laminar flow conditions

y	G	D/D_ε	A_c/A_ε	Λ_ε	$f_D(\varepsilon)Re_D$
1.0	1.8621	1.6796	1.4311	4.0368	63.648
1.5	1.4480	1.6245	1.2240	3.2302	50.930
2.0	1.2716	1.6163	1.1358	2.9670	46.782
2.5	1.1810	1.6176	1.0905	2.8535	44.991
3.0	1.1288	1.6206	1.0644	2.7954	44.074
3.5	1.0961	1.6234	1.0480	2.7620	43.548
4.0	1.0743	1.6257	1.0372	2.7411	43.219
4.5	1.0592	1.6275	1.0296	2.7272	43.000
5.0	1.0482	1.6290	1.0241	2.7175	42.847
5.5	1.0400	1.6301	1.0200	2.7104	42.735
6.0	1.0337	1.6310	1.0168	2.7051	42.652
6.5	1.0288	1.6318	1.0144	2.7011	42.588
7.0	1.0249	1.6324	1.0124	2.6979	42.537
7.5	1.0217	1.6329	1.0108	2.6953	42.497
8.0	1.0191	1.6333	1.0096	2.6932	42.464
8.5	1.0169	1.6337	1.0085	2.6915	42.437
9.0	1.0151	1.6340	1.0076	2.6901	42.415
9.5	1.0136	1.6342	1.0068	2.6889	42.396
10.0	1.0123	1.6345	1.0061	2.6879	42.379
15.0	1.0055	1.6356	1.0027	2.6826	42.297
20.0	1.0031	1.6361	1.0015	2.6808	42.269
25.0	1.0020	1.6363	1.0010	2.6800	42.256
30.0	1.0014	1.6364	1.0007	2.6795	42.248
35.0	1.0010	1.6364	1.0005	2.6793	42.244
40.0	1.0008	1.6365	1.0004	2.6791	42.241
45.0	1.0006	1.6365	1.0003	2.6790	42.239
50.0	1.0005	1.6365	1.0002	2.6789	42.238
∞	1	$(\pi+2)/\pi$	1	$(\pi+2)^2/\pi^2$	42.232

so that

$$\left(\frac{D}{D_\varepsilon}\right)_{\min} = 1.61629. \tag{11}$$

Up to this point no restriction to laminar flow has been imposed and the results are equally valid for turbulent flow.

3. FRICTION FACTOR CORRELATION FOR LAMINAR FLOW

The presence of a tape of finite thickness in a smooth tube leaves essentially a circular segment duct for the extremal case of infinite tape pitch as is shown in Fig. 1. The product $f\ Re$ for fully developed laminar flow in a straight duct of semicircular cross-section is [4]

$$f\ Re = 15.767. \tag{12}$$

According to Sparrow and Haji-Sheikh [8] the variation of $f\ Re$ with respect to the angle ϕ is almost linear for the small range of ϕ usually encountered in twisted-tape flow. From Table 77 of Shah and London [4] then follows for $\delta \ll D$

$$(f\ Re)_\varepsilon = 15.767 - \frac{15.767-15.690}{\pi/6}\gamma$$

$$= 15.767 - 0.147\frac{\delta}{D}. \tag{13}$$

Equation (13) has been obtained by matching the theory with the analytically exact value of $f\ Re$ for the infinite tape pitch situation. No secondary flow, nor the effect of the different lengths of the streamlines, is taken into consideration and adjustments to account for each of these effects will be corrected for later on.

This effective flow concept may now be used to obtain a value for $(f\ Re)_D$ based on the internal diameter of the tube.

The value of f_D which is derived by means of the effective flow concept is designated by $f_D(\varepsilon)$ in order to distinguish it from the real value of f_D, which is to be obtained by means of a correction in accordance with the numerical results. Therefore

$$(f\ Re)_D \equiv f_D(\varepsilon)Re_D = \Lambda_\varepsilon(f\ Re)_\varepsilon \tag{14}$$

where

$$\Lambda_\varepsilon \equiv \frac{A_t D^2}{A_\varepsilon D_\varepsilon^2}. \tag{15}$$

For any tube and twisted-tape combination thus follows

$$f_D(\varepsilon)Re_D = \Lambda_\varepsilon(15.767 - 0.147\delta/D). \tag{16}$$

This result is demonstrated in Fig. 3 for the cases $y = \infty$ and 3 (both with $\delta = 0$) as straight lines parallel to the line representing the no-tape condition of equation (12). This model accurately predicts the friction factor in the region of mild twist and low Reynolds number. In the regions of higher twist and Reynolds number where the secondary flow becomes more pronounced, a modification has to be introduced in order to account for the secondary effects. The effective flow friction factor–Reynolds number product from equation (16) is presented in Fig. 4, which may serve as a utility chart for easy reference to the effective flow predictions.

4. NUMERICAL RESULTS

Numerical results for laminar twisted-tape flow with $\delta = 0$ are available from work by the present authors [5]. Based on the SIMPLE method of Patankar and Spalding [10] for parabolic flows, and incorporating the helical coordinate system introduced by Date [2], a

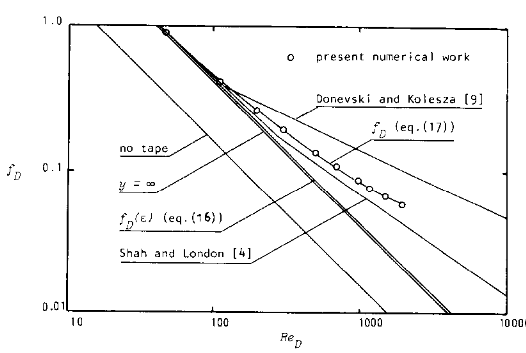

FIG. 3. Friction factor for fully developed twisted-tape flow with $y = 3$ and $\delta = 0$.

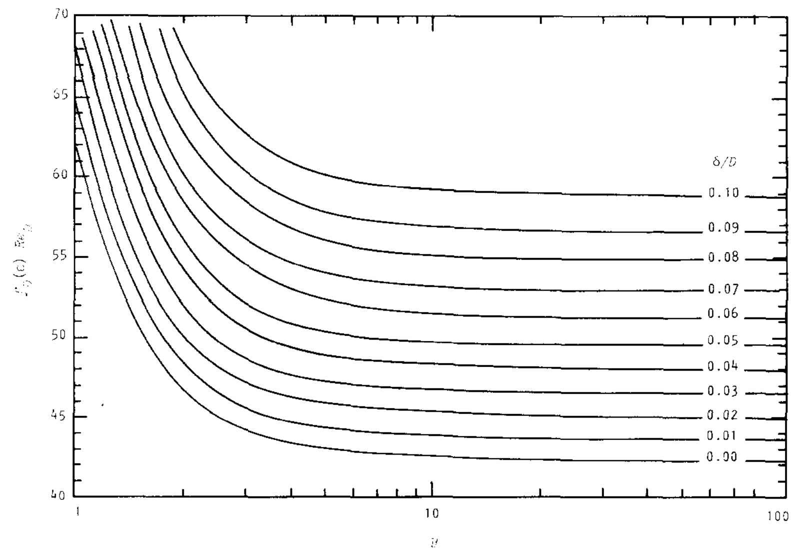

FIG. 4. Utility chart of effective flow factor $f_D(\varepsilon)Re_D$.

primitive variable solution procedure is used to predict developing flow in a smooth tube containing a twisted-tape insert with $\delta = 0$. Cross-sectionally a nonuniform 17×17 grid is employed with a converged solution at each cross-sectional plane being obtained iteratively. Axially marching integration is performed with increasing steplengths until a fully developed situation, defined by a fractional change criterion, is reached. The friction factors for these fully developed solutions are employed in this section.

In Table 2 and Fig. 5 are presented the results from a numerical parametric study with $\delta = 0$ to indicate the influence of twist ratio y and the Reynolds number Re_D on the friction factor for fully developed twisted-tape flow. The general trends are similar to the findings of Date [2] although more pronounced as will be clear from the results of this section. The method suggested by Churchill and Usagi [11] may be used to obtain a suitable correlation for the variations in the friction factor as is presented in Fig. 5.

The limiting value needed when $Re_D \to 0$ is aptly supplied by the concept of effective flow. Through the definition of a fictious asymptote when $y \to 0$ the following expression may be derived for the friction factor

$$f_D = f_D(\varepsilon)\left[1+\left(\frac{Re_D}{y^{1.3}}\right)^{1.5}\right]^{1/3}. \tag{17}$$

The success of this type of correlation is clear from Fig. 6 where this correlation is applied to the results of the present numerical work. In Fig. 3 a comparison of this

Table 2. Friction factor results for fully developed laminar twisted-tape flow

	Re_D				
y	50	200	700	1500	2000
∞_{an}	0.8446	0.2112	0.0603	0.0282	0.0211
∞	0.8415	0.2104	0.0601	0.0283	0.0212
	(0.996)	(0.996)	(0.997)	(1.004)	(1.005)
50	0.8418	0.2105	0.0602	0.0284	0.0216
	(0.997)	(0.997)	(0.998)	(1.009)	(1.023)
20	0.8435	0.2109	0.0615	0.0311	0.0246
	(0.998)	(0.998)	(1.109)	(1.104)	(1.164)
10	0.850	0.2132	0.0666	0.0364	0.0297
	(1.003)	(1.006)	(1.101)	(1.288)	(1.402)
5	0.874	0.226	0.0802	0.0479	0.0404
	(1.020)	(1.055)	(1.310)	(1.677)	(1.886)
3	0.937	0.261	0.1043	0.066	0.0572
	(1.063)	(1.184)	(1.656)	(2.280)	(2.595)
2	1.074	0.333	0.1530	0.1022	—
	(1.148)	(1.424)	(2.290)	(3.277)	—

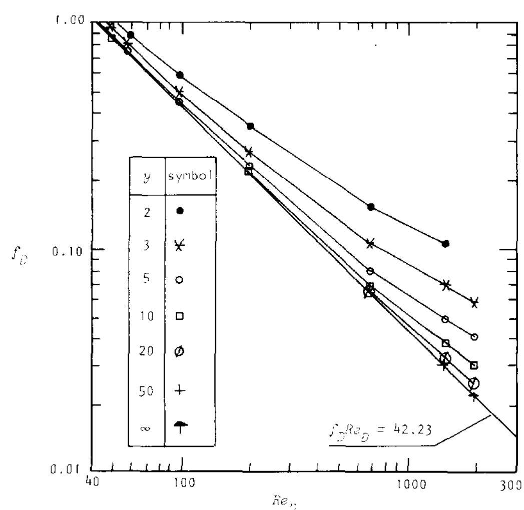

FIG. 5. Numerical values of friction factor for twisted-tape flow.

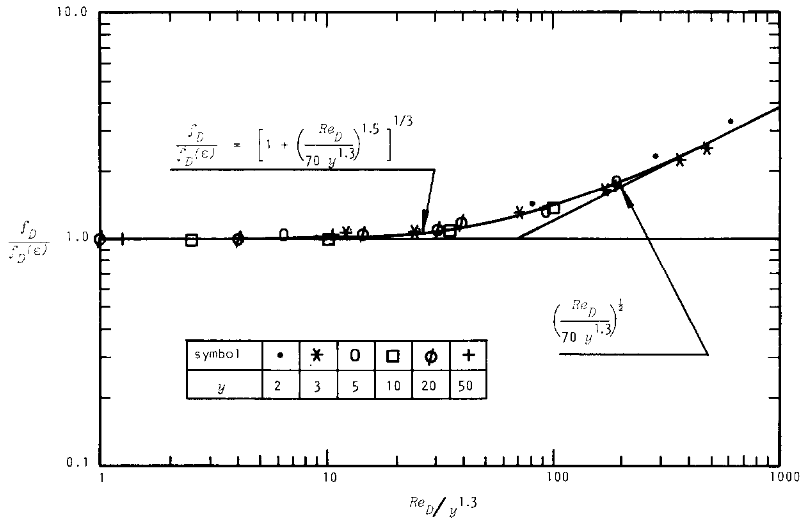

FIG. 6. Correlation of numerical results for twisted-tape flow with $\delta = 0$.

result with the predictions by Shah and London [4] and Donevski and Kolesza [9] is shown. In Table 3, a set of sequential equations is presented by which the factors of the correlative equation (17) may be obtained directly from the real tape and tube dimensions.

5. DISCUSSION

Equation (17) is the final result for predicting values of the friction factor f_D. The quantity $f_D(\varepsilon)$ may be obtained from the algorithm presented in Table 3 or it may be obtained from interpolation on the chart presented in Fig. 4.

It is clear from Fig. 5 that the curves representing lines of constant tape twist do not converge to one asymptote as $Re_D \to 0$. This behaviour led in the first instance to the conclusion that the only way of constructing a meaningful predictive measure is along the lines presented here. The method shows promise and is indeed accurate in the range of $Re_D/y < 100$. The exact coefficients for the high swirl corrective measure may need alteration when more accurate and substantial results become available. Although tape thickness is taken into account, there is a need for numerical work in this respect, since experiments are conducted with tapes of finite thickness.

In engineering practice twisted-tape flow is induced to enhance heat transfer effectivity of a system. The friction factor characteristics may then be greatly influenced by changes in temperature-dependent physical properties of the fluid. The numerical prediction in such cases can be done [1], but a suitable correlative equation has not yet been proposed. The present work, however, provides a suitable single equation on which future work in this direction may be based. This is to be preferred above the piecewise linear set of equations suggested by Shah and London [4].

The present results are based on numerical work over the laminar range $50 \leqslant Re_D \leqslant 2000$. This limitation, together with the restriction $y \geqslant 2$ on the tape twist, defines the range of applicability of equation (17).

Table 3. Step sequence for the calculation of the correlative function for the friction factor

$y = H/D$
$G = [1+(\pi/2y)^2]^{1/2}$
$A_t = \pi D^2/4$
$P_\varepsilon = 2D - 2\delta + \pi D/(2G)$
$A_\varepsilon = 2H^2(G-1)/\pi - D\delta$
$D_\varepsilon = 4A_\varepsilon/P_\varepsilon$
$\Lambda_\varepsilon = A_t D^2/(A_\varepsilon D_\varepsilon^2)$
$f_D(\varepsilon) = \Lambda_\varepsilon/Re_D \cdot (15.767 - 0.14706\delta/D)$
$f_D = f_D(\varepsilon)\,[1+(Re_D/(70y^{1.3}))^{1.5}]^{1/3}$

6. CONCLUSION

A new way of predicting friction factors for laminar twisted-tape flow is presented. It is based on suggestions by Nazmeev and Nikolaev [6] for heat transfer characteristics and may equally well be extended to turbulent flow characteristics. Tape thickness is taken into account in a simple manner. The present result may prove useful to engineers working on twisted-tape flows and also provides scope for more numerical and experimental research. To this end a chart is provided to minimize the effort needed by users of the procedure.

Research is now being undertaken to generalize this type of correlation to simultaneously developing flow and subsequently also turbulent flow. Heat transfer characteristics as such may also be predicted along similar lines as was indeed done by Nazmeev and Nikolaev [6]. Further results in this respect by the present authors will be published shortly.

REFERENCES

1. J. P. Du Plessis, Laminar flow and heat transfer in a smooth tube with a twisted-tape insert, Ph.D. thesis, University of Stellenbosch, Stellenbosch, South Africa (1982).
2. A. W. Date, Hydrodynamic characteristics of the fully-developed laminar flow in tubes containing twisted-tapes: a numerical study, Report No. HTS/70/10, Dept. of Mechanical Engineering (1970).
3. A. W. Date and J. R. Singham, Numerical prediction of friction and heat transfer characteristics of fully developed laminar flow in tubes containing twisted tapes, A.S.M.E., Paper 72-HT-17 (1972).
4. R. K. Shah and A. L. London, Laminar flow forced convection in ducts, in *Advances in Heat Transfer* (edited by F. I. Thomas and J. P. Hartnett), Suppl. 1. Academic Press, London (1978).
5. J. P. Du Plessis and D. G. Kröger, Numerical prediction of laminar flow and heat transfer in a tube with a twisted-tape insert, *Proc. 3rd Int. Conf. on Numerical Methods in Laminar and Turbulent Flow*, Seattle, U.S.A. (1983).
6. Yu. G. Nazmeev and N. A. Nikolaev, Correlation of experimental data on heat transfer in tubes with twisted-tape turbulence promotors, *Thermal Engineering*, USSR **27**(3), 151–152 (1980), translated from Teploénergetika **27**(3), 51–53 (1980).
7. E. V. Seymour, Fluid flow through tubes containing twisted tapes, *Engineer* **222** (5779), 634–642 (1966).
8. E. M. Sparrow and A. Haji-Sheikh, Laminar heat transfer and pressure drop in isosceles triangular, right triangular and circular sector ducts, *Trans. Am. Soc. Mech. Engrs*, Series C, *J. Heat Transfer* **87**, 426–427 (1965).
9. B. Donevski and J. Kolesza, Resistance coefficients for laminar and turbulent flow in helical shaped ducts (in Polish), *Archivum Termodynamiki: Spalania* **9**(3), 497–506 (1978).
10. S. V. Patankar and D. B. Spalding, A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows, *Int. J. Heat Mass Transfer* **15**, 1787–1805 (1972).
11. S. W. Churchill and R. Usagi, A general expression for the correlation of rates of transfer and other phenomena, *A.I.Ch.E. Jl* **18**(6), 1121–1128 (1972).

CALCUL DU FACTEUR DE FROTTEMENT POUR UN ECOULEMENT ETABLI LAMINAIRE AVEC RUBAN TORSADE

Résumé—Les caractéristiques du facteur de frottement sont nécessaires pour l'évaluation de l'efficacité de l'accroissement du transfert thermique par insertion d'un ruban torsadé dans un tube lisse. Des paramètres de l'écoulement sont introduits pour faciliter la formulation des résultats sur le facteur de frottement dans de tels cas. Un graphe utile est présenté pour l'estimation du produit facteur de frottement–nombre de Reynolds dans l'écoulement sur ruban torsadé. Des résultats d'une étude numérique paramétrique sur l'écoulement laminaire, à propriétés constantes avec ruban torsadé sont fournis et présentés graphiquement. Ces résultats sont utilisés pour construire une expression à partir de laquelle le facteur de frottement pour écoulement laminaire peut être obtenu. On prend en considération l'épaisseur finie du ruban.

BERECHNUNG DES REIBUNGSBEIWERTES FÜR VOLLSTÄNDIG AUSGEBILDETE LAMINARE STRÖMUNG IN EINEM ROHR, DAS EIN VERDRILLTES BAND ENTHÄLT

Zusammenfassung—Für die Berechnung des Wirkungsgrades bei der Verbesserung des Wärmeüberganges durch Einbau eines verdrillten Bandes in ein glattes Rohr ist die Kenntnis von Reibungsbeiwerten erforderlich. Es werden effektive Strömungsparameter eingeführt, um die Korrelation von Reibungsbeiwerten in solchen Fällen zu erleichtern. Ein Arbeitsdiagramm wird vorgestellt, in welchem das Produkt aus Reibungsbeiwert und Reynoldszahl für die Strömung in einem Rohr mit einem verdrillten Band abgelesen werden kann. Die Ergebnisse einer numerischen Parameterstudie über die laminare Strömung in einem Rohr mit einem verdrillten Band mit konstanten Fluideigenschaften werden beschrieben und ebenfalls graphisch dargestellt. Die Ergebnisse werden zur Aufstellung einer Korrelationsgleichung verwendet, aus welcher sich der Reibungsbeiwert für laminare Strömung ermitteln lässt. Die endliche Dicke des Bandes wird berücksichtigt.

РАСЧЕТ КОЭФФИЦИЕНТА ТРЕНИЯ ДЛЯ ПОЛНОСТЬЮ РАЗВИТОГО ЛАМИНАРНОГО ТЕЧЕНИЯ ТИПА СКРУЧЕННОЙ ЛЕНТЫ

Аннотация—Характеристики коэффициента трения необходимы для оценки эффективности интенсификации теплопереноса за счет использования в гладкой трубе вставки типа скрученной ленты. Для лучшего обобщения данных по коэффициенту трения в таких случаях вводятся эффективные параметры потока. В графическом виде представлена простая схема, по которой можно рассчитать коэффициент трения от числа Рейнольдса для течения типа скрученной ленты. Даны результаты параметрического численного исследования ламинарного закрученного потока с постоянными свойствами, которые также представлены в графическом виде. Эти результаты использованы для составления обобщенной зависимости, из которой можно получить характеристики коэффициента трения для ламинарного течения. Учитывается конечная толщина ленты.